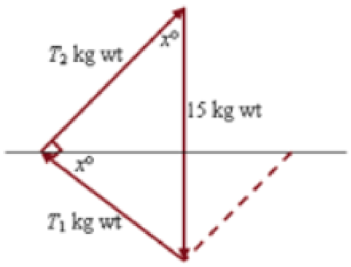
Solutions to short-answer questions

1 Note that the two strings form a 3- 4- 5 triangle. Draw the triangle of forces.

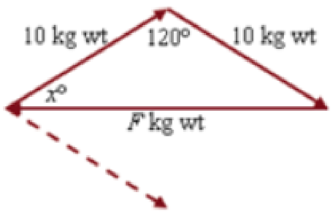


Note:

$$\sin x = \frac{6}{10} = \frac{3}{5}; \cos x = \frac{8}{10} = \frac{4}{5}$$

$$\begin{aligned} T_1 &= 15 \sin x \\ &= 15 \times \frac{3}{5} = 9 \text{ kg wt} \\ T_2 &= 15 \cos x \\ &= 15 \times \frac{4}{5} = 12 \text{ kg wt} \end{aligned}$$

2 Draw the triangle of forces.



Use the cosine rule.

$$\begin{aligned} F^2 &= 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos 120^\circ \\ &= 100 + 100 - 200 \times -\frac{1}{2} \\ &= 300 \\ F &= \sqrt{300} \\ &= 10\sqrt{3} \text{ kg wt} \end{aligned}$$

Since the triangle is isosceles,

$$\begin{aligned} x &= \frac{180 - 120}{2} \\ &= 30^\circ \end{aligned}$$

$10\sqrt{3}$ kg wt, at 150° to each 10 kg wt force.

3 The force exerted on the body by the plane will be perpendicular to the plane.

Resolve parallel to the plane, so the component this force will be zero.

The hypotenuse of the marked triangle is

$$\begin{aligned} h &= \sqrt{12^2 + 6^2} \\ &= \sqrt{180} = 6\sqrt{5} \text{ cm} \end{aligned}$$

If x is the angle of the plane to the horizontal,

$$\sin x = \frac{6}{6\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$\cos x = \frac{12}{6\sqrt{5}} = \frac{2}{\sqrt{5}}$$

Resolving,

$$T - 70\sin x = 0$$
$$T = 70\sin x$$
$$= 70 \times \frac{1}{\sqrt{5}}$$
$$= \frac{70\sqrt{5}}{5} = 14\sqrt{5}\text{ kg wt}$$

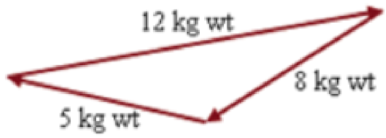
Resolving perpendicular to the plane,

$$N - 70\cos x = 0$$
$$N = 70\cos x$$
$$= 70 \times \frac{2}{\sqrt{5}}$$
$$= \frac{140\sqrt{5}}{5} = 28\sqrt{5}\text{ kg wt}$$

4 The force exerted on the body by the plane will be perpendicular to the plane.
Resolve parallel to the plane, so the component of this force will be zero.

$$F\cos 30^\circ - 15\sin 30^\circ = 0$$
$$\frac{F\sqrt{3}}{2} = 15 \times \frac{1}{2}$$
$$F = \frac{15}{\sqrt{3}}$$
$$= \frac{15\sqrt{3}}{3}$$
$$= 5\sqrt{3}\text{ kg wt}$$

5 Draw the triangle of forces and use the cosine rule.



$$\cos x = \frac{12^2 + 5^2 - 8^2}{2 \times 15 \times 5}$$
$$= \frac{105}{120} = \frac{7}{8}$$

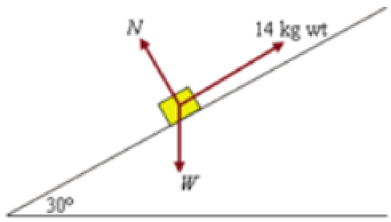
Since the required angle is $180^\circ - x$, the cosine is $-\frac{7}{8}$.

6
$$F\cos 30^\circ = 20$$
$$\frac{F\sqrt{3}}{2} = 20$$
$$F = \frac{20 \times 2}{\sqrt{3}}$$
$$= \frac{40\sqrt{3}}{3}\text{ kg wt}$$

7 Resolve parallel to the plane.

$$F - 15\sin 45^\circ = 0$$
$$F = 15\sin 45^\circ$$
$$= \frac{15\sqrt{2}}{2}\text{ kg wt}$$

8



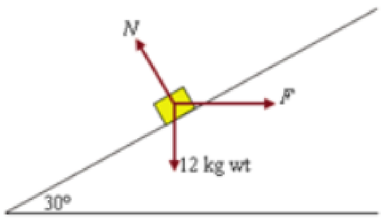
Resolve parallel to the plane.

$$\begin{aligned} W\sin 30^\circ - 14 &= 0 \\ W &= \frac{14}{\sin 30^\circ} \\ &= \frac{14}{0.5} = 28 \text{ kg wt} \end{aligned}$$

Resolve perpendicular to the plane.

$$\begin{aligned} N - 28\cos 30^\circ &= 0 \\ N &= 28\cos 30^\circ \\ &= \frac{28\sqrt{3}}{2} \\ &= 14\sqrt{3} \text{ kg wt} \end{aligned}$$

9



Calculate F by resolving parallel to the plane.

$$\begin{aligned} F\cos 30^\circ - 12\sin 30^\circ &= 0 \\ \frac{F\sqrt{3}}{2} &= 12 \times \frac{1}{2} \\ F &= 6 \times \frac{2}{\sqrt{3}} \\ &= \frac{12\sqrt{3}}{3} \\ &= 4\sqrt{3} \text{ kg wt} \end{aligned}$$

Solutions to multiple-choice questions

1 **E**

$$\begin{aligned} 50\cos 60^\circ &= 50 \times \frac{1}{2} \\ &= 25 \text{ N} \end{aligned}$$

2 **C** Use Pythagoras' theorem.

$$\begin{aligned} \text{Resultant} &= \sqrt{5^2 + 4^2} \\ &= \sqrt{41} \text{ kg wt} \end{aligned}$$

3 **E** Resolve perpendicular to the plane.

$$\begin{aligned} N - 20\cos 30^\circ &= 0 \\ N &= 20\cos 30^\circ \\ &= 20 \times \frac{\sqrt{3}}{2} \\ &= 10\sqrt{3} \text{ kg wt} \end{aligned}$$

4 **A** Resolve parallel to the plane.

$$\begin{aligned} F - 20\sin 30^\circ &= 0 \\ F &= 20\sin 30^\circ \\ &= 20 \times \frac{1}{2} \\ &= 10 \text{ kg wt} \end{aligned}$$

5 **C** For the particle to be in equilibrium,

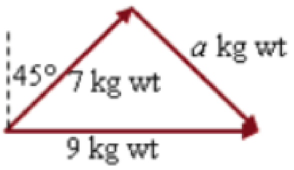
B must equal the sum of the forces on A and C.

$\therefore B = A\cos 60^\circ + C\cos 30^\circ$

(since $180 - 150 = 30$).

As this is true, $\mathbf{C}$ cannot be true.

6 **B**



Since the forces are perpendicular,

$$\begin{aligned} a^2 + 7^2 &= 9^2 \\ a^2 &= 81 - 49 \\ &= 32 \\ a &= \sqrt{32} = 4\sqrt{2} \end{aligned}$$

7 **B** The angle between the forces when they are head to tail will be 120°.

Use the cosine rule.

$$\begin{aligned} F^2 &= 20^2 + 20^2 - 2 \times 20 \\ &\quad \times 20 \times \cos 120^\circ \\ &= 400 + 400 - 800 \times -\frac{1}{2} \\ &= 1200 \\ F &= \sqrt{1200} \\ &= 20\sqrt{3} \text{ kg wt} \end{aligned}$$

8 **A** The angle between the forces when they are head to tail will be 120°.

Use the cosine rule.

$$\begin{aligned} F^2 &= 300^2 + 200^2 - 2 \times 300 \times 200 \times \cos 120^\circ \\ &= 90\,000 + 40\,000 - 120\,000 \times -\frac{1}{2} \\ &= 190\,000 \\ F &= \sqrt{190\,000} \\ &= 100\sqrt{19} \text{ kg wt} \end{aligned}$$

9 **C**
$$\begin{aligned} R &= \sqrt{16^2 + 30^2} \\ &= \sqrt{1156} \\ &= 34 \text{ kg wt} \end{aligned}$$

10 **B** The forces will be at right angles to each other.

$$\begin{aligned} a^2 + 8^2 &= 12^2 \\ a^2 &= 144 - 64 \\ &= 80 \\ a &= \sqrt{80} = 4\sqrt{5} \end{aligned}$$